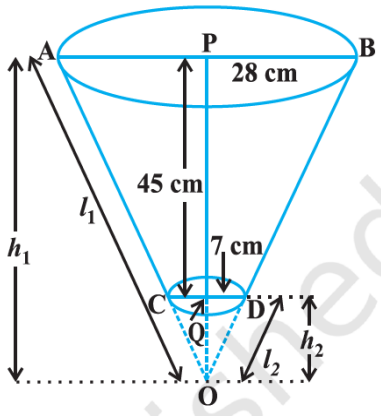
We have discussed about frustum of a cone and formulae used for the calculation of its curved surface area, total surface area and volume. Consider the problem.

Ex- 1.The radii of the ends of a frustum of a cone 45 cm high are 28 cm and 7 cm. Find its volume, curved surface area and the total surface area. (Use $\pi = \frac{22}{7}$)



Bigger Cone (AOB)
Radius, r_1 = 28 cm
Height, h_1 = 45 +h_2

Smaller Cone (COD)
Radius, r_2 = 7 cm
Height, h_2

Consider similar triangles ΔBOP and ΔDQP

$$\Delta BOP \sim \Delta DQP$$

$$\frac{BP}{DQ} = \frac{PO}{QO} \text{ and } \frac{BP}{DQ} = \frac{BO}{DO}$$

$$\frac{28}{7} = \frac{h1}{h2} \text{ and } \frac{28}{7} = \frac{l1}{l2}$$

$$h_1 = 60 \text{ cm and } h_2 = 15 \text{ cm}$$

$$l_1 = \sqrt{{h_1}^2 + {r_1}^2} = \sqrt{60^2 + 28^2} = \sqrt{3600 + 784} = \sqrt{4384} = 66.20 \text{ cm(approx.)}$$

$$l_2 = \sqrt{{h_2}^2 + 2^2} = \sqrt{15^2 + 7^2} = \sqrt{225 + 49} = \sqrt{274} = 16.55 \text{ cm(approx.)}$$

Curved Surface Area of Frustum = $\pi r_1 l_1 - \pi r_2 l_2$

= $\pi(r_1 l_1 - r_2 l_2)$

= $\frac{22}{7}(28\ x\ 66.20 - 7\ x\ 16.55)$

= 5461.5 cm^2

Total Surface Area of Frustum = C S A of frustum + $\pi {r_1}^2 + \pi {r_2}^2$

= 5461.5 cm^2 + $\frac{22}{7}$ x 28^2 + $\frac{22}{7}$ x 7^2

= 5461.5 cm^2 + 2464 cm^2 + 154 cm^2

= 8079.5 cm^2

The volume of the frustum of cone(ABDC) is the difference of the volume of cone(AOB) and cone (COD).

Volume of the frustum of cone(ABDC) = Volume of cone(AOB) – Volume of cone (COD).

Volume of the frustum of cone(ABDC) = $\frac{1}{3}\pi {r_1}^2 h_1 - \frac{1}{3}\pi {r_2}^2 h_2$

= $\frac{1}{3}\pi({r_1}^2 h_1 - {r_2}^2 h_2)$

= $\frac{1}{3}\ x\ \frac{22}{7} x\ (28^2 x\ 60 - 7^2 x\ 15)$

= $\frac{1}{3}\ x\ \frac{22}{7}\ x\ (28^2 x\ 60 - 7^2 x\ 15)$

= 48150 cm^3

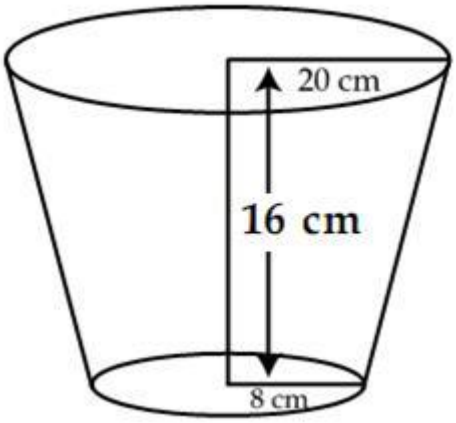
Note: C S A , T S A and Volume of Frustum of cone can be obtained using formulae.

CSA = $\pi r_1 l_1 + \pi r_2 l_2$; **T SA =** $\pi r_1 l_1 + \pi r_2 l_2 + \pi {r_1}^2 + \pi {r_2}^2$

Volume = $\frac{1}{3}\pi h({r_1}^2 + {r_2}^2 + r_1 r_2)$

Ex- 2.The radii of the lower and upper ends of a bucket in the form of frustum of a cone 16 cm high are 8 cm and 20 cm. Find the area of the metal sheet required to make the bucket and the capacity of the bucket.

(Use π = 3.14)



Frustum of cone

r_1 = 20 cm, r_2 = 8 cm , h = 16 cm

$$l^2 = h^2 + (r_1 - r_2)^2$$

$$l^2 = 16^2 + (20 - 8)^2 \text{ , } l^2 = 256 + 144 \text{ , } l^2 = 400 \text{ , } l = 20cm$$

Area of metal sheet required = $\pi(r_1 + r_2)l + \pi({r_1}^2 + {r_2}^2)$

$$= \pi(20 + 8)20 + \pi(20^2 + 8^2)$$

$$= 560\pi + 464\pi$$

$$= 1024\pi$$

$$= 1024 \, x \, 3.14$$

$$= 3215.36 \, cm^2$$

Capacity of the bucket = $\frac{1}{3}\pi h({r_1}^2 + {r_2}^2 + r_1 r_2)$

$$= \frac{1}{3} \, x \, 3.14 \, x \, 16 \, x(\, 20^2 + 8^2 + 20 \, x \, 8 \,)$$

$$= 10449.92 \, cm^3 = 10450 \, cm^3 (approx.)$$